Solutions to REVIEW OF TRIG SKILLS: MATH 112 **(Trigonometry and Conic Sections: 12/08ss)**

Objective II:

1. $\boxed{\begin{array}{l}\frac{\pi}{6}, \frac{\pi}{4}, 60°, 90°, \pi, 270°, 360° \\ \frac{2\pi}{3}, \frac{5\pi}{6}, 135°, 225°, \frac{7\pi}{4}, -150°, -315°\end{array}}$

2. a. $\boxed{2.02 \text{ radians}}$ b. $\boxed{48°42'}$

Objective III:

3. a. $\boxed{\frac{4}{5}}$ b. $\boxed{\frac{4}{3}}$ c. $\boxed{\frac{3}{5}}$ d. $\boxed{\frac{5}{3}}$ e. $\boxed{\frac{3}{4}}$ f. $\boxed{\frac{5}{4}}$

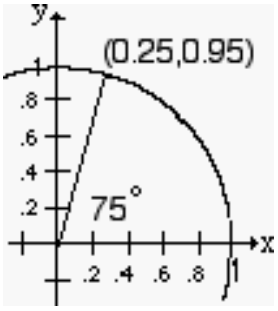
4. a. $\tan 42° = \frac{9.4}{b} \rightarrow \boxed{b \approx 10.4m}$

b. $\sin 40° = \frac{a}{18.6} \rightarrow \boxed{a \approx 12.0m}$

c. $\cos 39° = \frac{12.3}{X} \rightarrow \boxed{X \approx 15.8m}$

Objective IV:

5. a. $\boxed{0.95}$ b. $\boxed{0.25}$ c. $\frac{0.95}{0.25} \approx \boxed{3.8}$



6. $\cos\frac{7\pi}{3} = \boxed{0.5}$

7. a. $\boxed{\text{amp} = \frac{1}{2}; \text{per} = \pi; \text{ps} = 0; \text{vs} = 4}$

b. $\boxed{\text{no amp}; \text{per} = \pi; \text{ps} = -\frac{\pi}{2}; \text{vs} = 0}$

c. $\boxed{\text{amp} = 4; \text{per} = \frac{2\pi}{3}; \text{ps} = \frac{\pi}{3}; \text{vs} = 0}$

8. a. $\text{amp} = 4; \text{per} = \frac{2\pi}{b} = 4\pi \rightarrow b = \frac{1}{2}$
$\rightarrow \boxed{y = 4\sin\left(\frac{1}{2}x\right)}$

b. $\text{amp} = \frac{6-(-2)}{2} = 4; \text{per} = \frac{2\pi}{b} = 4\pi$
$\rightarrow b = \frac{1}{2}; \text{ps} = \pi; \text{vs} = 2 \rightarrow$
$\boxed{y = 2 + 4\sin\left[\frac{1}{2}(x-\pi)\right]}$

9. a. $\text{amp} = \frac{83.7 - 29.0}{2} = 27.35;$
$\text{per} = \frac{2\pi}{12} = \frac{\pi}{6}; \text{ps} = 4; \text{vs} = 29.0 + \text{amp}$
$\boxed{y = 27.35\sin\left[\frac{\pi}{6}(x-4)\right] + 56.35}$

Objective V:

10. a. $= \frac{1}{\cos x} = \boxed{\sec x}$

b. $= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} - \cos^2 x = 1 - \cos^2 x = \boxed{\sin^2 x}$

c. $\frac{\frac{1}{\cos x} \cdot \cos x + \frac{\sin^2 x}{\cos^2 x}}{\cos^2 x} = \frac{1 + \frac{\sin^2 x}{\cos^2 x}}{\cos^2 x} =$

$\left[\frac{1 + \frac{\sin^2 x}{\cos^2 x}}{\cos^2 x}\right] \cdot \frac{\cos^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^4 x}$

$= \frac{1}{\cos^4 x} = \boxed{\sec^4 x}$

d. $2\cos^2 x + (1 - \cos^2 x) = \boxed{1 + \cos^2 x}$

11. a. $\cos^2\theta - \sin^2\theta = (1-\sin^2\theta) - \sin^2\theta$
$= 1 - 2\sin^2\theta$

b. $\cos\theta + \sin\theta\tan\theta = \cos\theta + \sin\theta \cdot \dfrac{\sin\theta}{\cos\theta}$
$= \dfrac{\cos^2\theta}{\cos\theta} + \dfrac{\sin^2\theta}{\cos\theta}$
$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta}$
$= \dfrac{1}{\cos\theta} = \sec\theta$

Objective VI:

12. a. $\boxed{2\sin 40^\circ \cos 40^\circ}$

b. $\boxed{\cos 70^\circ \cos 20^\circ + \sin 70^\circ \sin 20^\circ}$

c. $\dfrac{270^\circ}{2} < \dfrac{\theta}{2} < \dfrac{360^\circ}{2} \rightarrow 135^\circ < \dfrac{\theta}{2} < 180^\circ$
$\rightarrow \cos\dfrac{\theta}{2}$ is neg $\rightarrow \boxed{\cos\dfrac{\theta}{2} = -\sqrt{\dfrac{1+\cos\theta}{2}}}$

13. a. $\sin^2\theta + \cos^2\theta = 1 \rightarrow \left(\dfrac{2}{3}\right)^2 + \cos^2\theta = 1$
$\rightarrow \dfrac{4}{9} + \cos^2\theta = 1 \rightarrow \cos^2\theta = \dfrac{5}{9}$
$\rightarrow \cos\theta = \pm\dfrac{\sqrt{5}}{3}$, and since θ is located in quadrant II, $\cos\theta < 0 \rightarrow \boxed{\cos\theta = -\dfrac{\sqrt{5}}{3}}$

b. $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\dfrac{2}{3}\right)\left(-\dfrac{\sqrt{5}}{3}\right) \rightarrow$
$\boxed{\sin 2\theta = -\dfrac{4\sqrt{5}}{9}}$

c. $\sin\dfrac{\theta}{2} = \sqrt{\dfrac{1-\dfrac{-\sqrt{5}}{3}}{2}} = \sqrt{\dfrac{1+\dfrac{\sqrt{5}}{3}}{2} \cdot \dfrac{3}{3}} \rightarrow$
$\boxed{\sin\dfrac{\theta}{2} = \sqrt{\dfrac{3+\sqrt{5}}{6}}}$

d. $\tan 2\theta = \dfrac{\sin 2\theta}{\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$
$= \dfrac{2\left(\dfrac{2}{3}\right)\left(-\dfrac{\sqrt{5}}{3}\right)}{\left(-\dfrac{\sqrt{5}}{3}\right)^2 - \left(\dfrac{2}{3}\right)^2} = \dfrac{-\dfrac{4\sqrt{5}}{9}}{\dfrac{5}{9} - \dfrac{4}{9}}$
$= \dfrac{-\dfrac{4\sqrt{5}}{9}}{\dfrac{1}{9}} = \boxed{-4\sqrt{5}}$

Objective VII:

14. a. $\boxed{30^\circ}$ b. $\boxed{-45^\circ}$ c. $\boxed{\dfrac{5\pi}{6}}$

15. a. $\boxed{23.4^\circ}$ b. $\boxed{74.5^\circ}$ c. $\boxed{0.6}$

16. $\cos A = \dfrac{6.9}{15.8} \rightarrow A = \cos^{-1} 0.4367 \rightarrow$
$\boxed{A \approx 64.1^\circ}$

Objective VIII:

17. a. $\sin x(\cos x - \sin x) = 0$
$\rightarrow \sin x = 0$ or $\cos x - \sin x = 0 \rightarrow$
$\sin x = 0 \rightarrow x = 0$ or $x = \pi$
$\cos x - \sin x = 0 \rightarrow \cos x = \sin x \rightarrow$
$1 = \dfrac{\sin x}{\cos x} \rightarrow 1 = \tan x \rightarrow x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$
$\boxed{x = 0, \dfrac{\pi}{4}, \pi, \dfrac{5\pi}{4}}$

b. $\tan^2 x = 3 \rightarrow \tan x = \pm\sqrt{3} \rightarrow$
$\boxed{x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}}$

c. $2\cos 2x = \sqrt{3} \rightarrow \cos 2x = \dfrac{\sqrt{3}}{2} \rightarrow$
$2x = \dfrac{\pi}{6} + 2\pi k \rightarrow x = \dfrac{\pi}{12} + \pi k$ OR
$2x = \dfrac{11\pi}{6} + 2\pi k \rightarrow x = \dfrac{11\pi}{12} + \pi k \rightarrow$

18. a. $\boxed{x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}}$

b. $\boxed{x \approx 0.68, 2.88, 3.25}$

b. $\boxed{x \approx 0.39, 1.57}$

Objective IX:

19. $\frac{\sin 31°}{x} = \frac{\sin 51°}{22.4} \rightarrow \boxed{x \approx 14.8 \text{ m}}$

20. $x^2 = 8.7^2 + 12.3^2 - 2(8.7)(12.3)\cos 100°$

$\boxed{x \approx 16.3 \text{in.}}$

21. $a^2 = 23.1^2 + 17.9^2 - 2 \cdot 23.1 \cdot 17.9 \cos 27°$

$a \approx 10.825 \rightarrow \frac{\sin 27°}{10.825} = \frac{\sin C}{23.1} \rightarrow \boxed{C \approx 104.3°}$

Note: *C* has to be largest angle (opposite largest side), so must be obtuse.

Objective X:

22. a.

θ	0	$\pi/4$	$2\pi/3$
R	1	1.59	4

23. a. one possibility: $\boxed{(r,\theta) = (13, 4.32)}$

b. $\boxed{\left(3\sqrt{3}, -3\right)}$

Objective XI:

24. a. $\sqrt{(-2)^2 + (6)^2} = 2\sqrt{10} \approx \boxed{6.32}$

25. a. $\boxed{\sqrt{34}\left(\cos 149° + i \sin 149°\right)}$

b. $\boxed{-2.12 + 2.12i}$

26. a. $-6 + 3i - 10i + 5i^2 = \boxed{-11 - 7i}$

b. $\left(\frac{3+5i}{-2+i}\right) \cdot \frac{-2-i}{-2-i} = \frac{-1-13i}{4+1} = \boxed{-\frac{1}{5} - \frac{13}{5}i}$

27. a. $5 \cdot 6\left[\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)\right] =$

$\boxed{30\left(\cos\frac{7\pi}{12} + i \sin\frac{7\pi}{12}\right)}$

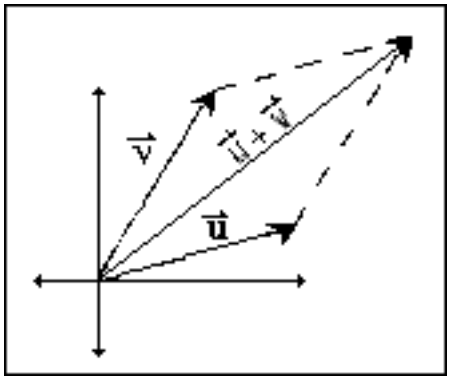
b. $\frac{5}{6}\left[\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)\right] =$

$\boxed{\frac{5}{6}\left(\cos\frac{\pi}{12} + i \sin\frac{\pi}{12}\right)}$

Objective XII:

28. a. i. $\boxed{\langle 5, 2 \rangle}$ ii. $\boxed{\langle -1, -8 \rangle}$ iii. $\boxed{\langle 1, -11 \rangle}$

29. a.



30. a. $\boxed{\text{mag} = \sqrt{58} \approx 7.62, \theta = 1.98 \text{radians}}$

b. $\boxed{\text{mag} = 2\sqrt{5} \approx 4.47, \theta = 5.82 \text{radians}}$

31.

$\langle 50\cos 30°, 50 \sin 30° \rangle + \langle 40\cos 45°, 40 \sin 45° \rangle$

$= \langle 43.3, 25 \rangle + \langle 28.3, 28.3 \rangle = \langle 71.6, 53.3 \rangle$

$\text{Mag} = \sqrt{71.6^2 + 53.3^2} = \boxed{89.3 \text{pounds}}$

$\tan\theta = \frac{53.3}{71.6} \rightarrow \boxed{\theta \approx 36.7°}$

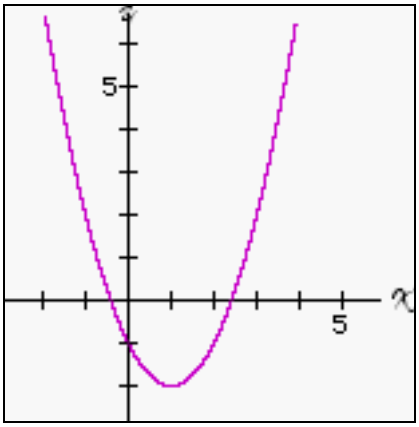
32. a. $\boxed{-2\vec{i} + 5\vec{j}}$

b. $\langle 4\cos 90°, 4 \sin 90° \rangle = \langle 0, 4 \rangle = \boxed{4\vec{j}}$

Objective XIII:

33. a.

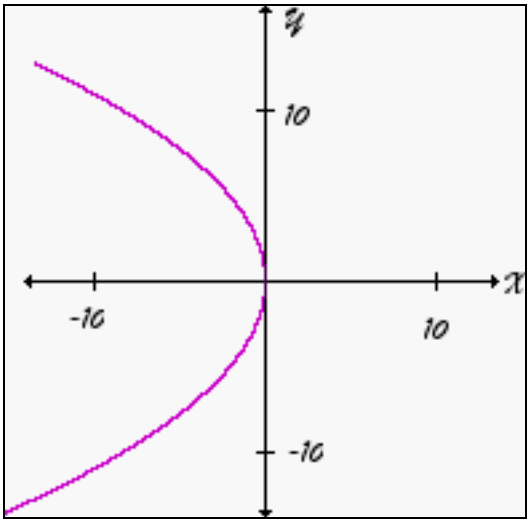
t	0	1	2	3
x	1	2	3	4
y	–2	–1	2	7



34. $t = x - 1 \rightarrow \boxed{y = (x-1)^2 - 2}$

Objective XIV:

35. a.



b. $4p = -12 \rightarrow p = -3 \rightarrow \boxed{\text{focus}: (-3,0)}$

c. directrix : $\boxed{x = 3}$

36. $x^2 = 4py \rightarrow \boxed{x^2 = 16y}$

37. a. $2^2 + b^2 = 3^2 \rightarrow b = \sqrt{5}$

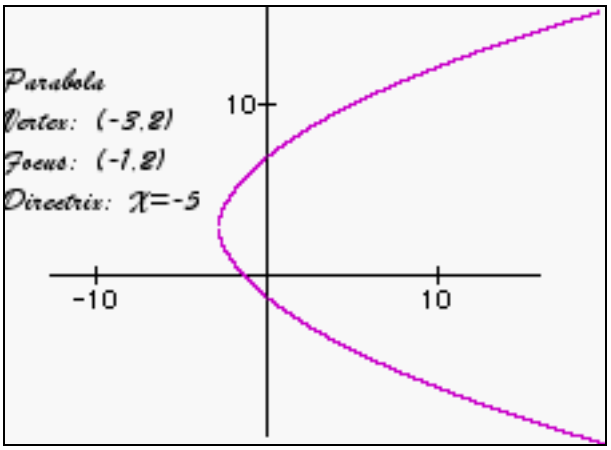
$$\frac{x^2}{\sqrt{5}^2} + \frac{y^2}{3^2} = 1 \rightarrow \boxed{\frac{x^2}{5} + \frac{y^2}{9} = 1}$$

b. $3^2 + b^2 = 4^2 \rightarrow b = \sqrt{7}$

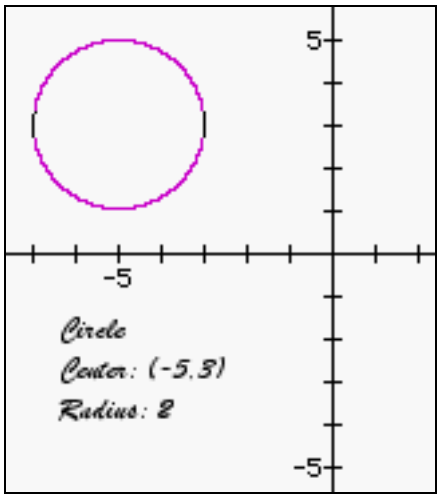
$$\frac{x^2}{3^2} - \frac{y^2}{\sqrt{7}^2} = 1 \rightarrow \boxed{\frac{x^2}{9} - \frac{y^2}{7} = 1}$$

38. $\boxed{y = \pm\frac{3}{2}x}$

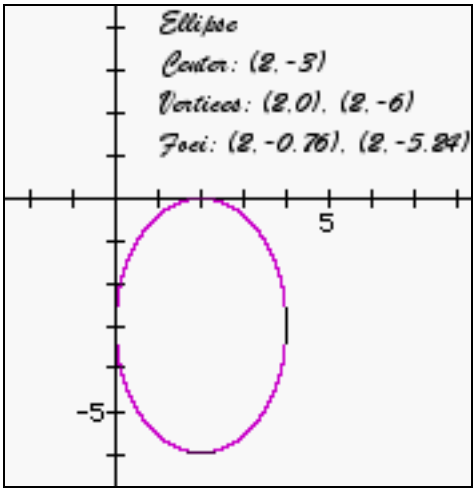
39. a.



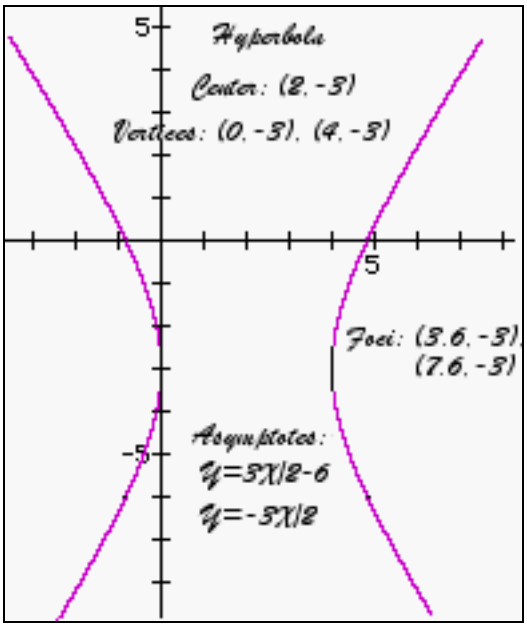
b.



c.



d.



40. $9(x^2-4x)+4(y^2+2y)=-31$

$9(x^2-4x+4)+4(y^2+2y+1)=-31+40$

$9(x-2)^2+4(y+1)^2=9$

$$\boxed{(x-2)^2+\frac{4(y+1)^2}{9}=1}$$

41. $$\frac{(x-4)^2}{4^2}+\frac{(y-5)^2}{2^2}=1$$

$$\boxed{\frac{(x-4)^2}{16}+\frac{(y-5)^2}{4}=1}$$

Objective XV:

42. $\tan 70° = \dfrac{A}{200} \rightarrow A \approx 549.5$ ft

$\tan 70.5° = \dfrac{B}{200} \rightarrow B \approx 564.8$ ft

Flagpole : $564.8 - 549.5 = \boxed{15.3 \text{ ft}}$

43. $$\frac{1\text{mi}}{2\text{min}} \cdot \frac{1\text{min}}{60\text{sec}} \cdot \frac{5280\text{ft}}{1\text{mi}} \cdot \frac{1\text{rev}}{\pi(2.5)\text{ft}} =$$

$\boxed{5.6 \text{ rev/sec}}$

Objective XVI

44. a. $x \approx -2.24, x = 0, x \approx 2.24$

b. $x \approx -1.29, x = 0$

45. $x \approx 0.59, 2.55, 4.25, 5.18$

46. The function has a maximum value of 5 when $x \approx 1.57$.