REVIEW SHEETS
COLLEGE ALGEBRA
MATH 111

A Summary of Concepts Needed to be Successful in Mathematics

The following sheets list the key concepts that are taught in the specified math course. The sheets present concepts in the order they are taught and give examples of their use.

WHY THESE SHEETS ARE USEFUL –

- To help refresh your memory on old math skills you may have forgotten.
- To prepare for math placement test.
- To help you decide which math course is best for you.

HOW TO USE THESE SHEETS –

- Students who successfully review spend from four to five hours on this material. We recommend that you cover up the solutions to the examples and try working the problems one by one. Then, check your work by looking at the solution steps and the answer.

KEEP IN MIND –

- These sheets are not intended to be a short course. You should use them simply to help you determine at what skill level in math you should begin study. For many people, the key to success and enjoyment of learning math is in getting started at the right place. You will, most likely, be more satisfied and comfortable if you start onto the path of math and science by selecting the appropriate beginning stepping stone.

I. Maintain, use, and expand the skills and concepts learned in previous mathematics courses.

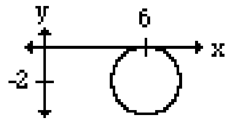
If you need to refresh your skills in intermediate algebra and geometry, please purchase the review materials for those courses.

II. Apply the midpoint formula, distance formula, properties of lines, and equations of circles to the solution of problems from coordinate geometry.

1. Write the standard equation of a circle whose center is $(4,-1)$ and whose radius is 2.

2. Sketch a graph (without using a calculator) of $x^2 + y^2 = 16$ and of $(x-3)^2 + (y+2)^2 = 16$.

3. Write the equation of the circle shown:



4. Find the standard equation of the circle whose diameter has endpoints $(-5,8)$ and $(-3,-5)$.

5. Find the equation of the line that passes through $(3,-7)$ and is perpendicular to the line $6x + 2y = 8$.

6. Write the equation of the

   a. vertical line passing through $(-2,5)$.

   b. horizontal line through $(-2,5)$.

7. Write the equation of the line passing through $(-3,5)$ and $(9,11)$.

III. Use and apply the concepts, language, notation, and evaluation of functions, including input-output ideas, domain, range, increasing, decreasing, maximum values, minimum values, symmetry, odd, even, composition of functions, and inverses.

A. Determine from a description, table, or graph whether the relation is a function.

8. At a particular point on Earth, is temperature a function of the time of day? Is the time of day a function of the temperature?

9. For which of the following tables and graphs is y a function of x? For which is x a function of y?

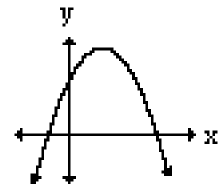
a.

x	–4	0	4
y	10	10	10

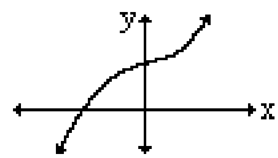
b.

x	–2	0	3
y	6	4	8

c.



d.



B. Given a geometric situation, define a function for a given quantity.

10. Write an equation giving the area, A, of a square as a function of the length of a side, s. Explain how you know that A is a function of s.

11. Write the equation of the circumference, C, of a circle as a function of its radius, r.

C. Evaluate using function notation.

12. Given the function $f(x) = 3x - 2x^2$, evaluate and simplify:

   a. $f(-3)$

   b. $f(2x)$

   c. $f(-3+h) - f(-3)$

   d. $\dfrac{f(x+h) - f(x)}{h}$

D. Add, subtract, multiply, divide, and compose functions.

13. Given $f(x) = 3x - 2x^2$ and $g(x) = \dfrac{2}{x}$, find:

    a. $(f+g)(2)$

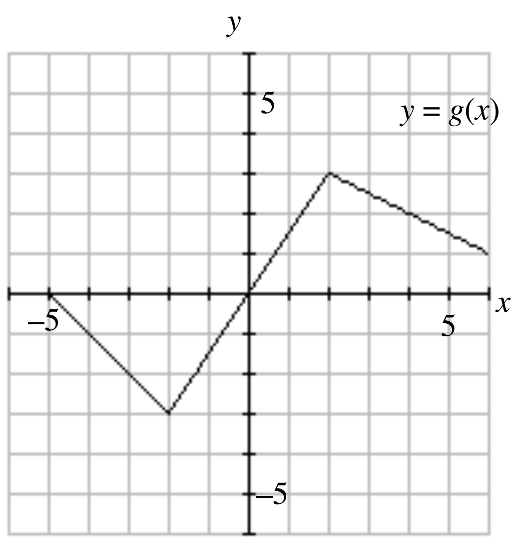
    b. $(fg)(2)$

    c. $(f \circ g)(2)$

    d. $(f \circ g)(x)$. Find the domain of $f \circ g$.

14. A function $y = f(x)$ is shown in the table, and a function $y = g(x)$ is shown in the graph.

x	$y = f(x)$
–3	–2
–1	2
1	4
4	–1
6	3



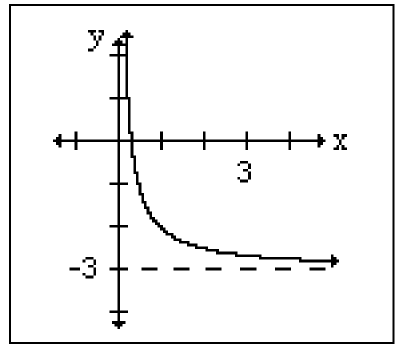
If possible, find:

a. $(f-g)(4)$ b. $\left(\dfrac{f}{g}\right)(-3)$

c. $(g \circ f)(-1)$ d. $(f \circ g)(0)$

E. Identify the domain and range of a function from tables, equations, and graphs.

15. Give the domain and range of the function whose graph is shown:



16. Give the domain and range of each function: [A hand sketched graph might help.]

    a. $f(x) = |x+2| - 3$

    b. $g(x) = \dfrac{3}{x+2}$

    c. $q(x) = \sqrt{x+3}$

    d. $m(x) = \log_4(x+3)$

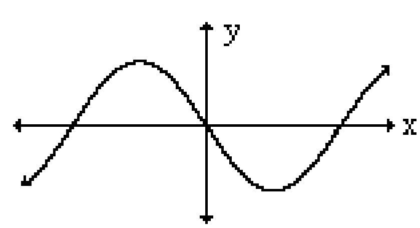
    e. $n(x) = e^{x-1}$

17. Explain in sentences why the domain of $q(x) = \sqrt{x+3}$ is not all the real numbers.

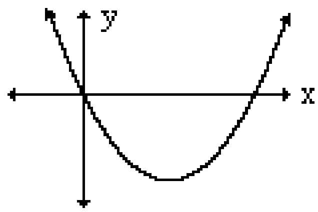
F. Determine whether a function, $y = f(x)$ is even (graph is symmetric about the y-axis) or odd (graph is symmetric about the origin) when given its graph or equation.

18. Which of the graphs below depict a function that is even? odd? neither?

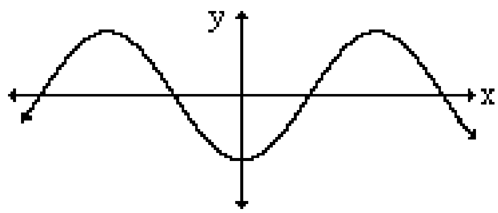
    a.



b.



c.



19. Use the algebraic test to determine if these functions are odd, even, or neither.
[Odd: $f(-x) = -f(x)$; Even: $f(-x) = f(x)$]

a. $f(x) = \dfrac{x^2}{x^2 - 5}$

b. $g(x) = x^3 + x^2$

c. $h(x) = x - 2x^5$

d. $q(x) = 6$

G. Create graphs given information about the function but not its equation.

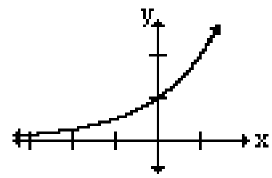
20. Sketch the graph of an odd function (anything you can dream up!) with the following properties:

Domain is $[-6,6]$
Range is $[-1,1]$
Increasing over the interval $(-3,3)$
Decreasing over intervals $(-6,-3)$ and $(3,6)$

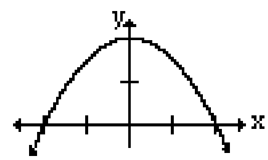
H. Determine equations and graphs of inverse functions.

21. For each function $y = f(x)$ below that has an inverse function, sketch a graph of that inverse.

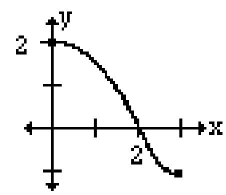
a.



b.



c.



22. If an inverse function in #21 a-c does not exist, describe how the domain of the original function might be restricted so an inverse would exist.

23. Determine algebraically whether $f(x) = \dfrac{2x+3}{5}$ and $g(x) = \dfrac{5x-3}{2}$ are inverses of each other.

24. Determine graphically whether the functions in #23 are inverses of each other.

25. Determine an equation for the inverse function of each of the functions:

a. $g(x) = x^3 + 2$

b. $f(x) = \dfrac{x}{x+2}$

c. $y = \sqrt{x-3}$

d. $h(x) = 5e^{x+3} - 2$

e. $y = \log_2(3x-1)$

I. Determine values of functions and inverse functions from tables, graphs, and equations.

26. Use the given table of values for $y = f(x)$ to determine the values.

x	$f(x)$
–2	–9
0	–1
2	7

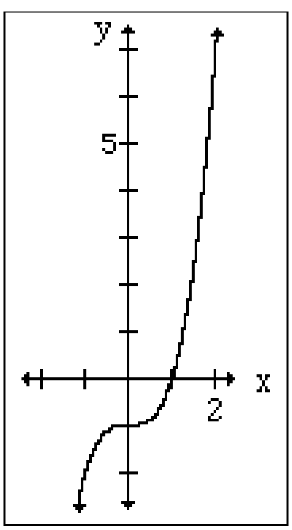
a. $f(2) = ?$

b. If $f(x) = -1$, then $x = ?$

c. $f^{-1}(-1) = ?$

d. $f^{-1}(7) = ?$

27. Use the graph of $y = f(x)$ to determine:



a. $f(1) = ?$

b. If $f(x) = -1$, then $x = ?$

c. $f^{-1}(7) = ?$

d. $f^{-1}(-1) = ?$

28. Use the equation $f(x) = x^3 - 1$ to determine:

a. $f(3) = ?$

b. If $f(x) = 7$, then $x = ?$

c. $f^{-1}(26) = ?$

d. $f^{-1}(63) = ?$

IV. Use substitution to create an equation defining one quantity as a function of another.

29. Write an equation giving the area, A, of a square as a function of its perimeter, P.

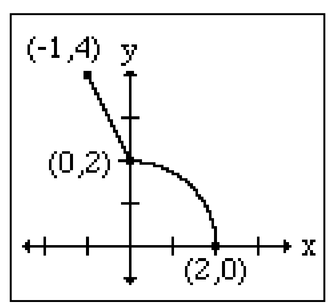
30. Write an equation giving the perimeter, P, of a square as a function of its area, A.

31. If the volume of an open box (i.e. no top) with a square base is 5 cubic feet, write the surface area, S, of the box as a function of x, the length of a side of the base.

32. The length of a rectangle is 3 more than twice the width, w. Write an equation giving the area, A, as a function of the width.

V. Apply principles of transformations (shifts, reflections, and stretches) to equations and graphs of functions.

33. The graph of $y = f(x)$ is given. Sketch the graph of each of the following:



a. $y = f(x+2)$

b. $y = f(x) + 2$

c. $y = -2f(x)$

d. $y = f(-x)$

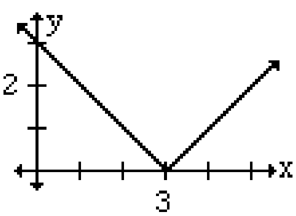
e. $y = f(2x)$

f. $y = f(x-2)-1$

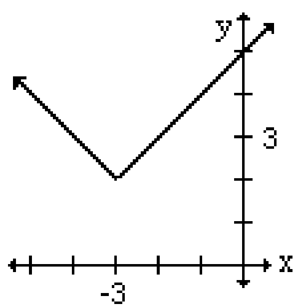
34. Using words such as reflection, shift, stretch, shrink, etc., give a step-by-step description of how the graph of $y = 3(x-2)^3$ can be obtained from the graph of a function $y = x^3$.

35. The following graphs are transformations of $y = |x|$. Write the equation of each graph. Verify your answers using your graphing calculator.

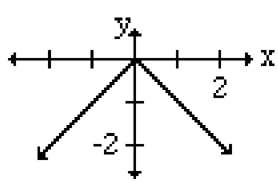
a.



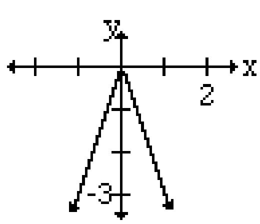
b.



c.



d.



36. Identify a basic function $y = f(x)$ and the transformations on that function to give $y = -\dfrac{4}{x-3}$.

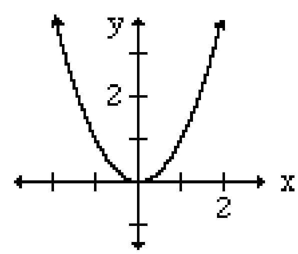
VI. Recognize, sketch, and interpret the graphs of the basic functions without the use of a calculator:

$f(x) = c,\quad x,\quad x^2,\quad x^3,\quad x^n,\quad \sqrt{x},\quad |x|,\quad e^x,$
$a^x \;(a>0),\quad \log_a x \;(a>1),\quad \ln x,\quad \dfrac{1}{x^n}$

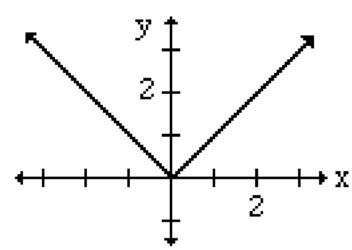
A. Recognize graphs of basic functions.

37. Write the equation for each basic graph:

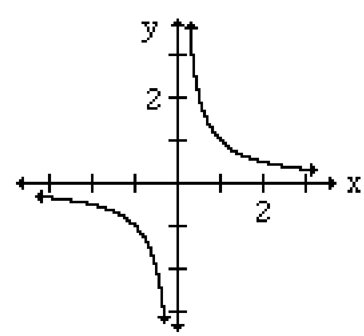
a.



b.



c.



B. Sketch the graphs of basic functions without the use of a graphing calculator.

38. Sketch a quick graph of each of the following basic functions. Be sure to have intercepts and other key points labeled.

a. $y = x^3$ b. $y = \log_3 x$

c. $y = \dfrac{1}{x^2}$

C. Interpret basic functions using their equations or graphs to identify: intervals where the function is increasing, decreasing or constant; local maxima and minima; asymptotes; and odd or even functions.

39. Determine if the following statements are true or false:

 a. $f(x) = x^2$ increases over its entire domain.

 b. $f(x) = e^x$ has no maximum or minimum value.

 c. $f(x) = \sqrt{x}$ has a maximum value of 0.

 d. $f(x) = 10^x$ has a vertical asymptote.

 e. $f(x) = \ln x$ has a vertical asymptote.

 f. $f(x) = \dfrac{1}{x^2}$ has both a vertical and a horizontal asymptote.

40. Answer the following questions using the list of functions:

$$f(x) = c,\quad x,\quad x^2,\quad x^3,\quad \sqrt{x},\quad |x|,\quad e^x,\quad a^x\ (a>1),\quad \log_a x\ \ (a>1),\quad \ln x,\quad \frac{1}{x},\quad \frac{1}{x^2}$$

 List the functions that:

 a. have a domain of $(-\infty,\infty)$.

 b. are increasing over their entire domain.

 c. have a local maximum .

 d. have a local minimum.

 e. have a range of $[0,\infty)$.

 f. have a range of $(-\infty,\infty)$.

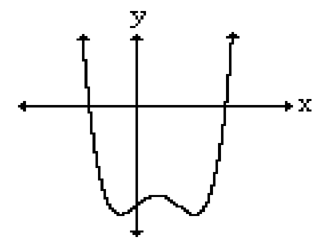
 g. have a vertical asymptote.

 h. have a horizontal asymptote.

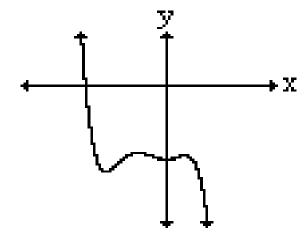
VII. Identify and apply properties of polynomial functions.

41. Which of the following graphs are possible graphs for a polynomial function of degree two, three, four, five, six, or seven?

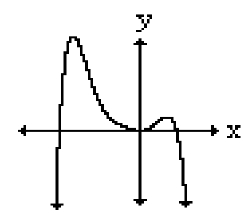
 a.



 b.

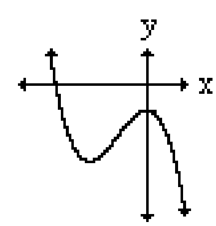


 c.



42. Which of the graphs above are possible for the graph of a 6^{th} degree polynomial function with a leading coefficient of –2?

43. Here is a graph of a polynomial function: Your friend thinks it might be the graph of a fifth degree polynomial function. Could your friend be right? Explain why or why not.



44. The zeros of a third degree polynomial function are 0, 2, –3. Write a possible equation for the function.

45. Three of the zeros of a fourth degree polynomial equation are $1, -1, 2i$. Write the equation for the function if $f(0) = 8$.

VIII. Solve nonlinear systems of equations algebraically and graphically.

46. Solve algebraically: $\begin{cases} x + y = 1 \\ x^2 + y^2 = 13 \end{cases}$

47. Solve graphically: $\begin{cases} x^2 + y^2 = 16 \\ (x+2)^2 + y^2 = 25 \end{cases}$

48. A circle is centered at the origin and has radius 4. Find the intersection of the circle and the line with slope –3 and y-intercept 2.

49. Use your calculator to graph $y = x^2$ and $y = 2^x$ on the same screen.

a. Identify all of the points of intersection.

b. Identify intervals where $x^2 < 2^x$.

c. Identify intervals where $x^2 > 2^x$.

IX. Identify and apply properties of rational functions with and without a calculator.

50. Identify the domain, y-intercept, and x-intercept(s) of $f(x) = \dfrac{2x-6}{x^2-16}$.

51. By hand, sketch the graph of $f(x) = \dfrac{2x-6}{x^2-16}$.

52. Identify the asymptotes of $f(x) = \dfrac{x^2}{2x^2-6}$. What happens to the value of the function, $f(x)$, as x increases without bound? As x decreases without bound?

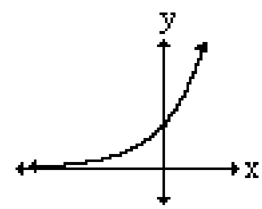
53. Write the equation of a rational function which has an oblique asymptote.

54. Create the equation for a function which has a vertical asymptote of $x = 3$ and a horizontal asymptote of $y = 5$. Now fix your equation so the function also has an x-intercept of $(2,0)$.

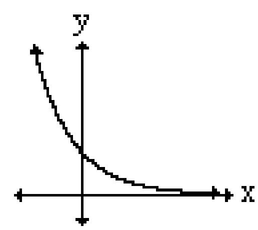
X. Identify and apply properties of exponential and logarithmic expressions and functions.

55. Which of the following graphs could be an exponential function (basic function $y = a^x$ or a transformation)?

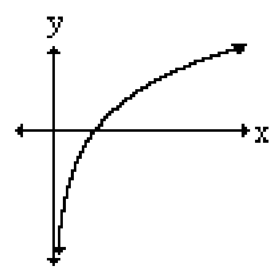
a.



b.



c.



56. Which of the graphs above could be of a logarithmic function (basic function $y = \log_b x$ or a transformation)?

57. By hand, sketch a graph of each of the following:

a. $y = 5^{x+3}$

b. $y = 5^x + 3$

c. $y = -\log_5 x$

d. $y = 2\log_5 x$

58. Expand as the sum of logs: $\log\sqrt{\dfrac{x}{x+7}}$

59. Solve these equations algebraically:

   a. $5^{2x+7} = 125$

   b. $10^{2x} \cdot 10^{5} = 1000$

   c. $\log_4(x-5) = 2$

   d. $x = \log_5 \sqrt[3]{25}$ (evaluate)

   e. $\log_4(x^2-25) - \log_4(x-5) = 3$

60. Solve these equations both algebraically and graphically:

   a. $5^{2x+7} = 60$

   b. $\ln(2x-5) = 2.63$

   c. $\log(x^2-25) - \log(x-5) = 3$

61. Money is invested at 8% compounded continuously. How much must be invested to have $10,000 in six years?

62. How much must be invested if the money above is compounded quarterly?

63. The function $P(t) = 3000(1.025)^t$ models the growth in population of a small town. In this model, P stands for the population, and t stands for the number of years since 2000. Use the model to predict the year that the population will first reach 8,000. Solve both algebraically and graphically.

64. There are 40 grams of radioactive substance at one point in time. After two hours, only 30 grams of the substance are radioactive. What is the half-life of the substance? (note: This is very unstable stuff!)

XI. Analyze a function by interpreting its graph, using a graphing calculator.

65. Graph $f(x) = x^5 - 3x^3 - x^2 - 4x - 1$ with your graphing calculator, and use the graph to answer these questions:

   a. Over what intervals is the function increasing? (Round to tenths.)

   b. Over what intervals is the function decreasing?

   c. Identify any local maxima and minima.

   d. For what value of x is $f(x)$ a maximum within the x-axis interval $(-5,4)$?

66. Graph the function $f(x) = \dfrac{3x^2-5}{x+2}$ with your graphing calculator.

   a. What is the domain of the function?

   b. What are the x-intercepts, if any?

   c. What are the y-intercepts, if any?

   d. Identify by equation any vertical, horizontal, or oblique asymptotes.

   e. What happens to the value of the function as x closely approaches –2 from the right?

   f. What happens to the value of the function, as x decreases without bound?

67. Use your graphing calculator to describe completely the important behavior of the polynomial function

$$f(x) = 5x^3 - 20x^2 + 2x - 1.$$

XII. Translate a set of numerical data into graphical form, choose a function (linear, power (x^n), exponential, logarithmic, or logistic) to model the data and interpret the implications of the model.

68. Here are five sets of input/output data.

x	–3	–2	–1	0	1	2	3
$f(x)$	4.5	2	0.5	0	0.5	2	4.5

x	–3	–2	–1	0	1	2	3
$g(x)$	–1	1	3	5	7	9	11

x	–3	–2	–1	0	1	2	3
$h(x)$	.125	0.25	0.5	1	2	4	8

x	–3	–2	–1	0	1	2	3
$r(x)$	8	12	32	53	74	94	98

x	–3	–2	–1	0	1	2	3
$s(x)$	0.25	0.50	0.75	1.00	1.25	1.50	1.75

a. Make a quick scatter plot for each set. What kind of function would fit each set of data best? (Try to do this first without using the curve fitting function of your calculator.) Choose one of these: linear, quadratic, exponential, logarithmic, or logistic. Explain your choice.

b. Without using a calculator, find the equation for all of the linear functions.

c. Using the curve fitting capability of your calculator, find the equation for $f(x)$, $g(x)$, $h(x)$, and $s(x)$.

d. Predict (approximate) the values for $f(x)$:
A. $f(2.3)$
B. $f(-4)$

69. You have discovered a previously unknown, unspoiled, unpopulated but lush Pacific island. Because you have accomplished this astounding feat, the powers of the world have decided to let you control the human population of the island as long as you allow a reasonable number of people to move or live there. You like people, and you like the natural environment of the island. Sketch a graph of how you might like the human population to grow. Decide whether the function which you graphed should be exponential, logarithmic, quadratic, linear, or logistic. Explain why you chose your graph and the type of function you did. There is not just one good answer. You can be creative and even humorous. Remember to apply the mathematics you have learned.

XIII. Translate word problems into mathematical expressions, solve the problems, and interpret the solutions.

Note: There are many examples of these types of questions in any college algebra or precalculus text. Several have already been indicated in this review. Below is another example.

70. Find all the values of x for which the distance between the points $(x,8)$ and $(-5,3)$ is 13.

XIV. Communicate ideas of college algebra through English statements and mathematical sentences.

For communicating through English statements, see especially problems 8, 10, 17, 43, 66, 67, 68, 69, 70, 71 in this review. One additional problem is given below. Mathematical sentences are used throughout most of this review.

71. In a certain situation, profit P is a function of the amount spent on advertisement, A. The graph of this function, $P = f(A)$, has a right side horizontal asymptote of $P = 30{,}000$. Write a sentence describing what this means To a person trying to make a profit in this situation.

XV. Use the language and skills of precalculus which are important for success in calculus.

Note: Almost everything in college algebra is important for calculus, but here are some specific examples:

A. Use function notation and interpret the difference quotient, which is the basis of the derivative in calculus.

72. On the graph of the function $f(x) = 2x^2 - 10$

 a. Locate the points associated with $x = 1$ and $x = 3$.

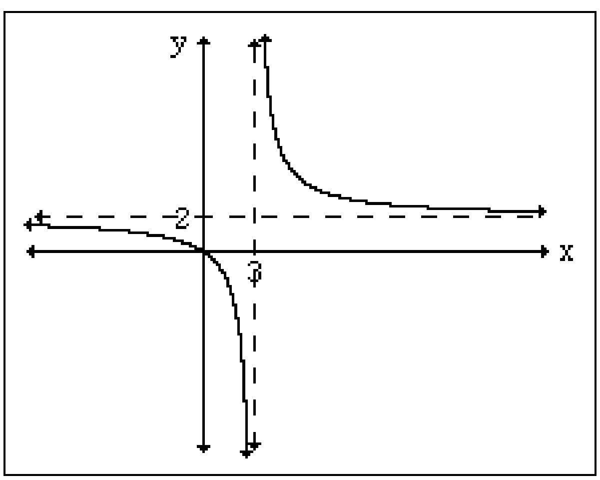
 b. Sketch the line through these two points.

 c. Calculate the slope of the line.

 d. Explain the relationship of this slope to the difference quotient $\dfrac{f(3) - f(1)}{3 - 1}$.

B. Use the language of precalculus that sets the stage for limits.

73. From the graph of $y = f(x)$ shown, answer true or false to each statement.



 a. As x becomes a very large positive number (x increases without bound), the value of the function, $f(x)$, approaches 2. $[\text{As} \quad x \to \infty, \quad f(x) \to 2]$

 b. As x approaches 3 from the right (x gets very close to 3 but is greater than 3), $f(x)$ increases without bound. $[\text{As} \quad x \to 3^+, \quad f(x) \to \infty]$

74. For the function in problem #73, describe what happens to the value of $f(x)$ as:

 a. x decreases without bound.

 b. x approaches 3 from the left.

XVI. Write and evaluate the notation of sequences and series including n^{th} terms, summations, and factorials.

75. Write the first four terms of a sequence if $a_n = \dfrac{2n}{n+3}$.

76. If $a_n = (-1)^{n+1} \dfrac{3^n}{n!}$, find a_4.

77. Write the first four terms of a sequence if $a_1 = 3$, and $a_{k+1} = 2a_k + 1$.

78. Simplify:

 a. $\dfrac{10!}{8!}$

 b. $\dfrac{n!}{(n+1)!}$

 c. $\dfrac{(2n+1)!}{(2n)!}$

79. Write an expression for the n^{th} term of each sequence.

 a. $\dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dfrac{5}{6}, \ldots$

 b. $-1, 4, -9, 16, -25, \ldots$

80. Find the sums:

a. $\sum_{i=1}^{4}(2i+3)$

b. $\sum_{k=2}^{5}(k-1)(k+2)$

81. Write in sigma notation:

a. $\frac{1}{4}+\frac{3}{8}+\frac{7}{16}+\frac{15}{32}=$

b. $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}=$

XVII. Identify sequences as arithmetic, geometric or neither.

82. Determine if each sequence is arithmetic, geometric or neither:

a. 3, 6, 12, 24, . . .

b. 5, 9, 13, 17, . . .

c. 2, 5, 10, 13, . . .

83. Write the n^{th} term of each sequence:

a. 2, 4, 8, 16, . . .

b. –3, 2, 7, 12, . . .

c. 81, –27, 9, –3, . . .

84. Find the sum of the first ten terms of each sequence:

a. 8, 20, 32, 44, . . .

b. 1, 2, 4, 8, . . .

85. Find the sum of these infinite geometric sequences:

a. $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ ...$

b. 1 + 2 + 4 + 8 + . . .

86. Solve using sums of sequences: A stadium has 25 rows of seats. There are 15 seats in the first row, 16 seats in the second row, 17 seats in the third row, and so on. How many seats in all 25 rows?

87. A deposit of $50 is put into an account on the first of every month. The account earns 6% interest compounded monthly. The balance, *A*, at the end of 2 years is given by:

$$A = 50\left(1+\frac{0.06}{12}\right)^{1}+\cdots+50\left(1+\frac{0.06}{12}\right)^{24}$$

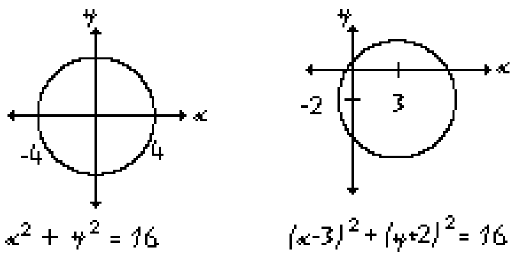
Find the value of *A*.

The End

ANSWERS

1. $(x-4)^2+(y+1)^2=4$

2.



3. $(x-6)^2+(y+2)^2=4$

4. $(x+4)^2+\left(y-\frac{3}{2}\right)^2=\frac{173}{4}$

5. $x-3y=24$

6a. $x=-2$

6b. $y=5$

7. $y=\frac{1}{2}x+\frac{13}{2}$, or $y=0.5x+6.5$

8. Yes, the temperature is a function of time. No, the time is not a function of the temperature since for a specific temperature, there could be different times.

9a. y is a function of x

9b. y is a function of x, and x is a function of y.

9c. y is a function of x

9d. y is a function of x, and x is a function of y

10. $A=s^2$. A is a function of s because each side length , s, there is only one area, A.

11. $C=2\pi r$

12a. -27

12b. $6x-8x^2$

12c. $15h-2h^2$

12d. $3-4x-2h$

13a. -1

13b. -2

13c. 1

13d. $\frac{6}{x}-\frac{8}{x^2}$; Domain: $x\neq 0$

14a. -3

14b. 1

14c. 3

14d. not possible

15. Domain: $x>0$ or $(0,\infty)$
Range: $y>-3$ or $(-3,\infty)$

16a. Domain: All Reals or $(-\infty,\infty)$
Range: $y\geq -3$ or $[-3,\infty)$

16b. Domain: $x\neq -2$; Range: $y\neq 0$

16c. Domain: $x\geq -3$; Range: $y\geq 0$

16d. Domain: $x>-3$; Range: All Reals

16e. Domain: All Reals; Range: $y>0$

17. For $q(x)$ to be a real number, $x+3\geq 0$, so $x\geq -3$.

18a. odd

18b. neither

18c. even

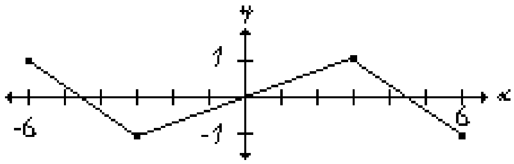
19a. even

19b. neither

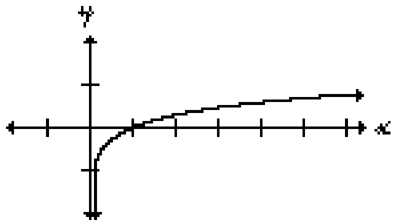
19c. odd

19d. even

20. One example:

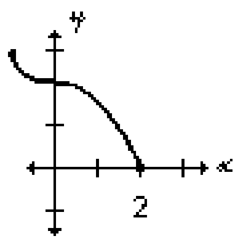


21a.



21b. This function has no inverse function.

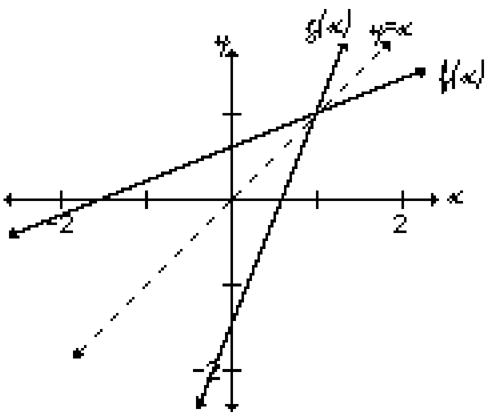
21c.



22. In 21b define domain as $x \geq 0$. Then the function will be one-to-one with an inverse function.

23. *f* and *g* are inverses because: $f(g(x)) = x$ and $g(f(x)) = x$.

24. The graphs of *f* and *g* are reflections across the line $y = x$, so *f* and *g* are inverse functions.



25a. $g^{-1}(x) = \sqrt[3]{x-2}$

25b. $f^{-1}(x) = \dfrac{2x}{1-x}$

25c. $y = x^2 + 3,\ x \geq 0$

25d. $h^{-1}(x) = \ln\left(\dfrac{x+2}{5}\right) - 3$

25e. $y = \dfrac{2^x + 1}{3}$

26a. 7

26b. 0

26c. 0

26d. 2

27a. 0

27b. 0

27c. 2

27d. 0

28a. 26

28b. 2

28c. 3

28d. 4

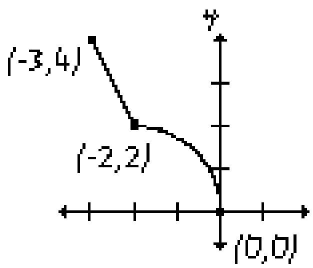
29. $A = \dfrac{P^2}{16}$

30. $P = 4\sqrt{A}$

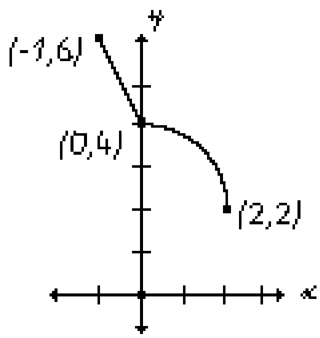
31. $S = x^2 + \dfrac{20}{x}$

32. $A = 2w^2 + 3w$

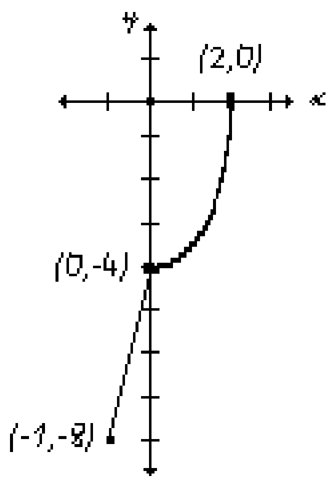
33a.



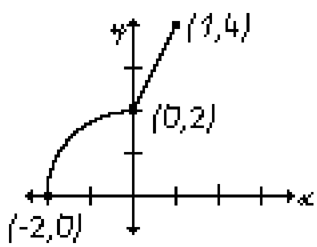
33b.



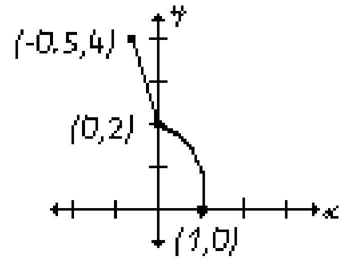
33c.



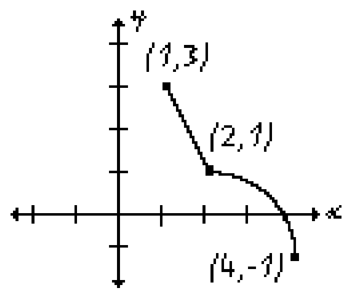
33d.



33e.



33f.



34. The graph of $y = x^3$ is shifted to the right 2 units and then each y value is multiplied by 3 (vertical stretch).

35a. $y = |x - 3|$

35b. $y = |x + 3| + 2$

35c. $y = -|x|$

35d. $y = -2|x|$

36. Basic Function: $y = \frac{1}{x}$

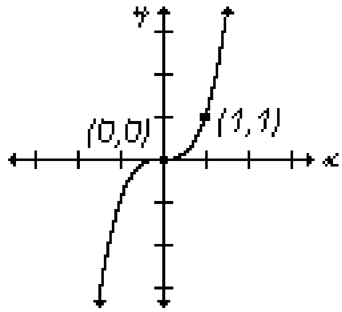
The function is shifted 3 units to the right, reflected in the x-axis, and then stretched vertically by a factor of 4.

37a. $f(x) = x^2$

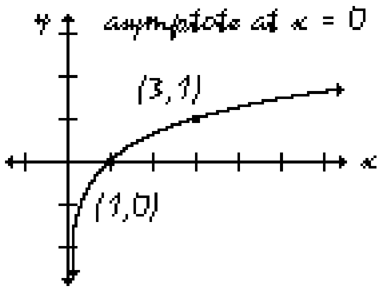
37b. $f(x) = |x|$

37c. $f(x) = \frac{1}{x}$

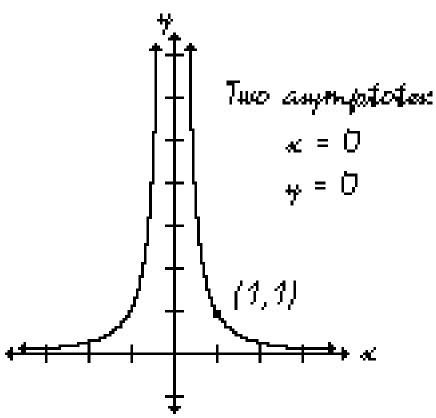
38a.



38b.



38c.



39a. False

39b. True

39c. False. Its minimum value is zero.

39d. False. It has a horizontal asymptote.

39e. True

39f. True

40a. $f(x) = c, x, x^2, x^3, |x|, e^x, a^x$

40b. $f(x) = x, x^3, \sqrt{x}, e^x, a^x\ (a > 1),$

$\log_a x\ (a > 1), \ln x, \dfrac{1}{x}$

40c. None

40d. $f(x) = x^2, |x|$

40e. $f(x) = x^2, \sqrt{x}, |x|$

40f. $f(x) = x, x^3, \log_a x, \ln x$

40g. $f(x) = \log_a x, \ln x, \dfrac{1}{x}, \dfrac{1}{x^2}$

40h. $f(x) = e^x, a^x, \dfrac{1}{x}, \dfrac{1}{x^2}$

41a. four, six

41b. five, seven

41c. four, six

42. "c" since extremities are both down.

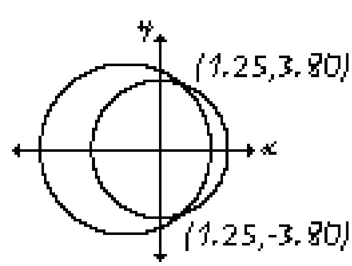
43. Yes, it might be a fifth degree polynomial since it has an end up and an end down.

44. $f(x) = x^3 + x^2 - 6x$
or any non-zero multiple

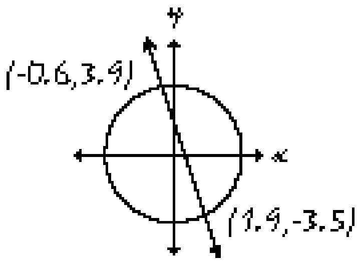
45. $f(x) = -2x^4 - 6x^2 + 8$

46. Solutions: $(-2,3); (3,-2)$

47.



48. $x^2 + y^2 = 16$ and $y = -3x + 2$



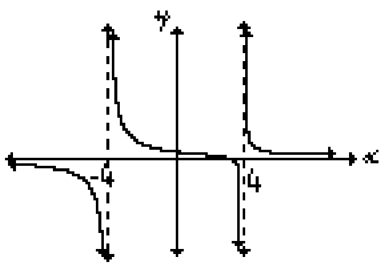
49a. $(-0.77,0.59);(2,4);(4,16)$

49b. $(-0.77,2)\cup(4,\infty)$

49c. $(-\infty,-0.77)\cup(2,4)$

50. Domain: $x \neq 4,-4$
y-intercept: $\left(0,\frac{3}{8}\right)$
x-intercept: $(3,0)$

51.



x-intercept = 3
y-intercept = 3/8
Asymptotes: $x = 4, -4$

52. Asymptotes:

$$x = \sqrt{3}$$
$$x = -\sqrt{3}$$
$$y = \frac{1}{2}$$

As x increases without bound, $f(x)$ approaches one-half.

As x decreases without bound, $f(x)$ approaches one-half.

53. The degree of the numerator must be one greater than the degree of the denominator, as in:

$$f(x) = \frac{x^3 + 2x}{x^2 - 4}$$

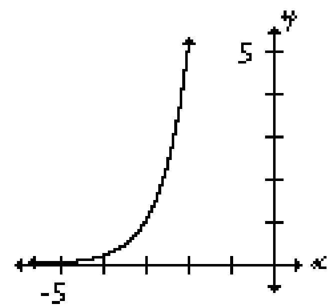
54. f has the required asymptotes; g has the asymptotes and x-intercept:

$$f(x) = \frac{5x}{x-3}; \qquad g(x) = \frac{5x-10}{x-3}$$

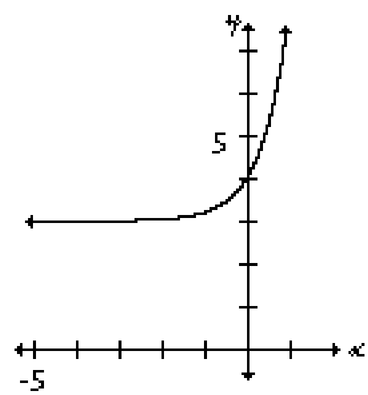
55. a, b

56. c

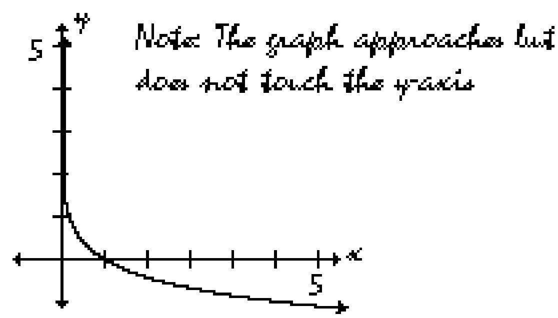
57a.



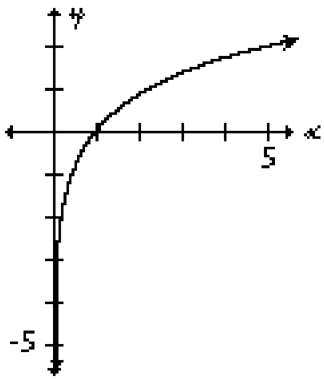
57b.



57c.



57d.



58. $\frac{1}{2}\log x - \frac{1}{2}\log(x+7)$

59a. $x = -2$

59b. $x = -1$

59c. $x = 21$

59d. $x = \frac{2}{3}$

59e. $x = 59$

60a. $x \approx -2.228$

60b. $x \approx 9.437$

60c. $x = 995$

61. $P \approx \$6188$

62. $P \approx \$6219$

63. year 2039

64. $t \approx 4.8$ hours for half – life

65a. $(-\infty,-1.4) \cup (1.5,\infty)$

65b. $(-1.4,1.5)$

65c. Local Maxima: 5.5
Local Minima: –11.8

65d. $x = -1.4$ at local maximum of 5.5

66a. Domain: $x \neq -2$

66b. $(1.3,0),(-1.3,0)$

66c. $(0,-2.5)$

66d. Vertical : $x = -2$
Oblique : $y = 3x - 6$

66e. $f(x) \to +\infty$

66f. $f(x) \to -\infty$

67.
y – intercept : $(0,-1)$
x – intercept : $(3.9,0)$
Local Min Point : $(2.62,-43.1)$
Local Max Point : $(0.06,-0.95)$
Increase Interval : $(-\infty,0.06) \cup (2.62,\infty)$
Decrease Interval : $(0.06,2.62)$
$f(x)$ is negative over $(-\infty,3.91)$
$f(x)$ is positive over $(3.91,\infty)$

68a. $f(x)$ is quadratic
$g(x)$ is linear
$h(x)$ is exponential
$r(x)$ is logistic
$s(x)$ is linear

68b. $g(x) = 2x + 5$
$s(x) = 0.25x + 1$

68c.
$f(x) = \frac{1}{2}x^2$
$g(x) = 2x + 5$
$h(x) = 2^x$
$s(x) = 0.25x + 1$

68d. A: $f(2.3) \approx 2.6$
B: $f(-4) \approx 8$

69. Many varied responses

70. $x=-17$ or $x=7$

71. A profit of $30,000 will be the most the company will earn from the advertisement, even if the company spends vast amounts on ads.

72a. Points: $(1,-8)$ and $(3,8)$

72b. ~sketch~

72c. Slope: $\frac{(8)-(-8)}{3-1}=\frac{16}{2}=8$

72d. The difference quotient gives the slope.

73a. True

73b. True

74a. $f(x)\to 2$

74b. $f(x)\to -\infty$

75. $\frac{1}{2},\frac{4}{5},1,\frac{8}{7}$

76. $-\frac{27}{8}$

77. 3, 7, 15, 31

78a. 90

78b. $\frac{1}{n+1}$

78c. $2n+1$

79a. $\frac{n+1}{n+2}$

79b. $(-1)^n n^2$

80a. $5+7+9+11=32$

80b. $4+10+18+28=60$

81a. $\sum_{k=1}^{4}\left(\frac{2^k-1}{2^{k+1}}\right)$

81b. $\sum_{j=1}^{4}\frac{(-1)^{j+1}}{3^j}$

82a. Geometric

82b. Arithmetic

82c. Neither

83a. Geometric: $a_n=2^n$

83b. Arithmetic: $a_n=5n-8$

83c. Geometric: $a_n=81\left(-\frac{1}{3}\right)^{n-1}$

84a. Arithmetic: $S_{10}=620$

84b. Geometric: $S_{10}=1023$

85a. $S=\frac{4}{3}$

85b. This sum is infinite.

86. Arithmetic: $15+16+17+\cdots+39$

$S_{25}=675$

87. Geometric: $S=\$1277.96$

<u>The End</u>