REVIEW SHEETS
TRIGONOMETRY
MATH 112

A Summary of Concepts Needed to be Successful in Mathematics

The following sheets list the key concepts which are taught in the specified math course. The sheets present concepts in the order they are taught and give examples of their use.

WHY THESE SHEETS ARE USEFUL –

- To help refresh your memory on old math skills you may have forgotten.
- To prepare for math placement test.
- To help you decide which math course is best for you.

HOW TO USE THESE SHEETS –

- Students who successfully review spend from four to five hours on this material. We recommend that you cover up the solutions to the examples and try working the problems one by one. Then, check your work by looking at the solution steps and the answer.

KEEP IN MIND –

- These sheets are not intended to be a short course. You should use them to simply help you determine at what skill level in math you should begin study. For many people, the key to success and enjoyment of learning math is in getting started at the right place. You will, most likely, be more satisfied and comfortable if you start onto the path of math and science by selecting the appropriate beginning stepping stone.

I. Use geometry, algebra, and graphing calculator skills from previous courses.

These skills are assumed in doing trigonometry. In a trigonometry course they might be reviewed in worksheets. Placement exams in geometry and algebra also cover many of these skills.

II. Move easily between degree and radian measure.

1. Know from memory the basic equivalencies and use them to calculate other equivalencies. Quickly complete the tables below without using a calculator:

Deg	Rad
$30°$	
$45°$	
	$\frac{\pi}{3}$
	$\frac{\pi}{2}$
$180°$	
	$\frac{3\pi}{2}$
	2π

Deg	Rad
$120°$	
$150°$	
	$\frac{3\pi}{4}$
	$\frac{5\pi}{4}$
$315°$	
	$-\frac{5\pi}{6}$
	$-\frac{7\pi}{4}$

2. Convert between degrees and radians for any given angle measure. Calculate the equivalent measures:

   a. $115°30' =$ ______ radians to the nearest hundredth

   b. 0.85 radians $=$ ____ ^{o} to the nearest minute

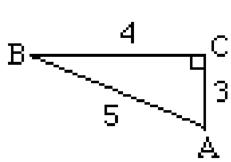
III. Identify and use the six trigonometric functions in right triangle applications.

3. Using the triangle shown, write a fraction for each of the following:

   a. $\sin A$ d. $\sec A$

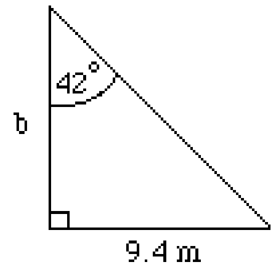
   b. $\tan A$ e. $\cot A$

   c. $\cos A$ f. $\csc A$

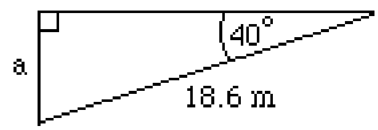


4. Calculate the requested length in each triangle below to the nearest tenth.

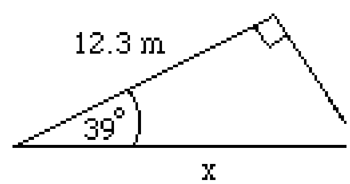
   a.



   b.



   c.



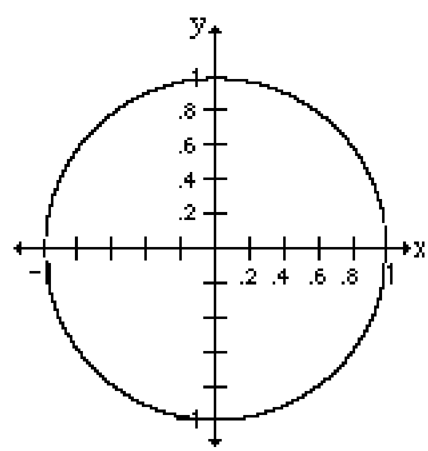
IV. Identify, apply, and interpret features of the equations and graphs of the six circular functions.

5. From the unit circle graph, give the approximate value of these:

   a. $\sin 75°$

   b. $\cos 75°$

   c. $\tan 75°$



6. On the rectangular coordinate system, sketch a graph of $y = \cos x$, and use it to determine the approximate value of $\cos\frac{7\pi}{3}$.

7. Give the amplitude, period, phase shift, and vertical shift for each equation. Then sketch a graph.

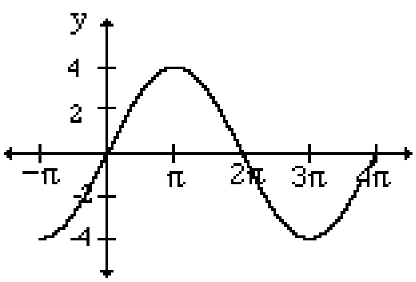
   a. $y = 4 + \frac{1}{2}\sin(2x)$

   b. $y = \tan\left(x + \frac{\pi}{2}\right)$

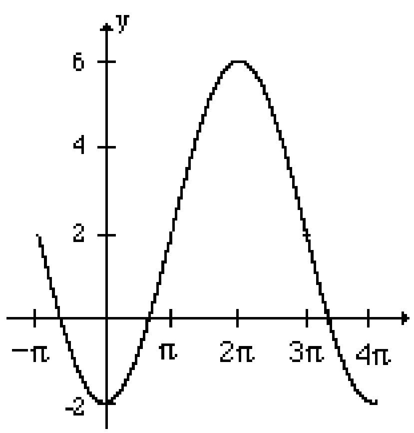
   c. $y = -4\cos(3x - \pi)$

8. Write an equation for each of these graphs using the sine function.

   a.



   b.



9. By hand, fit a sinusoidal function of the form $y = a\sin(bx + c) + d$ to a set of data.

   a. The table below gives the normal daily high temperatures for Chicago (F, in degrees Fahrenheit) for month t, with $t = 1$ corresponding to January. By hand, fit a sinusoidal function $y = F(t)$ to the data.

t	1	2	3	4	5	6
F	29.0	33.5	45.8	58.6	70.1	79.6

t	7	8	9	10	11	12
F	83.7	81.8	74.8	63.3	48.4	34.0

   b. Confirm that your function is a good fit by plotting a scatter plot in your calculator and graphing your function in the same window.

V. Recall and apply the basic trigonometric identities.

10. Simplify these expressions:

   a. $\dfrac{\sin^2 x + \cos^2 x}{\cos x}$

   b. $\tan x \cot x - \cos^2 x$

   c. $\dfrac{\sec x \cos x + \tan^2 x}{\cos^2 x}$

   d. $2\cos^2 x + \sin^2 x$

11. Verify the following identities:

   a. $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

   b. $\cos\theta + \sin\theta\tan\theta = \sec\theta$

VI. Use the sum, difference, double-angle and half-angle identities.

12. Fill in the blanks using the reference identities:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

   a. $\sin 80° = 2\sin\_\_\_\ \cos\_\_\_$

   b. $\cos(70° - 20°) = \cos\_\_\cos\_\_ + \sin\_\_\sin\_\_$

   c. for $270° < \theta < 360°$, $\cos\frac{\theta}{2} = \_\_\_\_\_$

13. Given $\sin\theta = \frac{2}{3}$ with $\frac{\pi}{2} < \theta < \pi$, use identities to find exact values for each of the following:

   a. $\cos\theta$

   c. $\sin\frac{\theta}{2}$

   b. $\sin 2\theta$

   d. $\tan 2\theta$

VII. Identify features of and use the three major inverse trigonometric functions.

14. Without a calculator give the value of these in the requested units:

a. $\sin^{-1}\left(\frac{1}{2}\right) = \_\_\_\_\ ^\circ$

b. $\tan^{-1}(-1) = \_\_\_\_\_\ ^\circ$

c. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \_\_\_\_$ radians

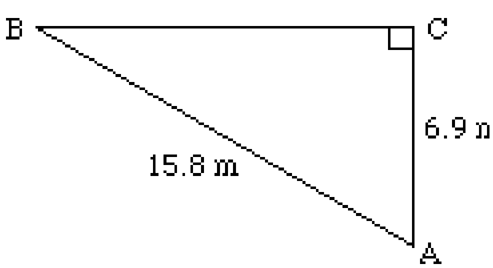
15. Using a calculator, give the value of these to the nearest tenth:

a. $\sin^{-1}(0.397) = \_\_\_\ ^\circ$

b. $\tan^{-1}(3.6) = \_\_\_\ ^\circ$

c. $\arccos(0.825) = \_\_\_$ radians

16. Calculate angle *A* to the nearest tenth of a degree.



VIII. Solve trigonometric equations analytically and with technology.

17. Solve analytically (use algebra and trigonometry but no calculator) for $0 \le x < 2\pi$:

a. $\sin x \cos x - \sin^2 x = 0$

b. $3 - \tan^2 x = 0$

c. $\sqrt{3} - 2\cos 2x = 0$

18. Solve using a graphing calculator for $0 \le x < 2\pi$:

a. $\cos 2x = 0.3x$

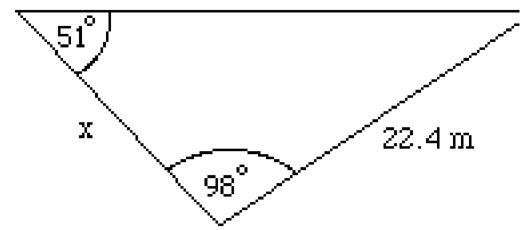
b. $3\sin x + 2 = 5 - 2\cos x$

IX. Apply the Law of Sines and Law of Cosines where appropriate.

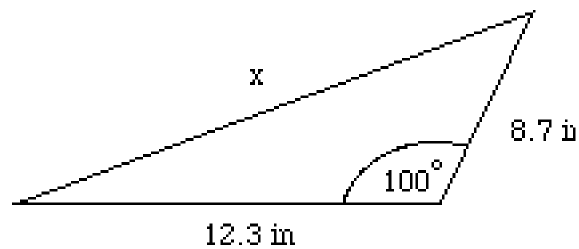
Law of Sines : $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines : $c^2 = a^2 + b^2 - 2ab\cos C$

19. Solve for the requested length to nearest tenth:



20. Solve for the requested length to nearest tenth:



21. In triangle ABC, $\angle A = 27°$, b = 17.9 feet, and c = 23.1 feet. Find $\angle C$.

X. Use polar coordinates and polar equations and transform them to rectangular form and back.

22. Plot points given in polar form and plot points from equations given in polar form.

a. Given the polar equation $r = 3 - 2\cos\theta$, complete the table and plot the points:

θ	r
0	
$\pi/4$	
$2\pi/3$	

23. Convert coordinates from rectangular to polar coordinates and vice versa.

a. Write polar coordinates for the rectangular coordinates $(-5, -12)$.

b. Write the rectangular coordinates for the polar coordinates $\left(6, -\frac{\pi}{6}\right)$.

XI. Use complex numbers in standard form and in polar form (optional).

24. Calculate the magnitude of a complex number.

    a. Calculate $|-2+6i|$

25. Switch between forms of complex numbers - standard form: $a+bi$ to polar form: $r(\cos\theta + i\sin\theta)$.

    a. Write $-5+3i$ in polar form.

    b. Write $3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ in standard form.

26. Add, subtract, multiply and divide complex numbers in standard form.

    a. $(3+5i)(-2+i)$ b. $\dfrac{3+5i}{-2+i}$

27. Multiply and divide complex numbers in polar form.

    a. $\left[5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]\left[6\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]$

    b. $\dfrac{5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}{6\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}$

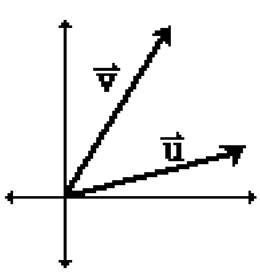
XII. Solve problems using vector notation.

28. Compute with vectors in component form.

    a. Given $\vec{u} = \langle 2,-3\rangle$ and $\vec{v} = \langle 3,5\rangle$, calculate
       i. $\vec{u} + \vec{v}$
       ii. $\vec{u} - \vec{v}$
       iii. $2\vec{u} - \vec{v}$

29. Sketch a vector which is the sum of given vectors in graphic form.

    a. Given vectors $\vec{u}$ and $\vec{v}$, sketch $\vec{u} + \vec{v}$:



30. Calculate the magnitude (length) and direction angle of a vector $(0 \le \theta < 2\pi)$.

    a. Calculate the magnitude and direction angle of the vector $\langle -3,7\rangle$.

    b. Calculate the magnitude and direction angle of the vector $\langle 4,-2\rangle$.

31. Calculate the resultant of two vectors given their magnitudes and direction angles.

    a. Two ropes are attached to a handle on a box. One rope is being pulled with a force of 50 pounds at a $30°$ angle to the horizontal. The other rope is being pulled with a force of 40 pounds at a $45°$ angle to the horizontal. Calculate the magnitude and direction angle of the resultant force.

32. Write a vector in the form $a\vec{i} + b\vec{j}$.

    a. Write $\langle -2,5\rangle$ in the form $a\vec{i} + b\vec{j}$.

    b. Write the $a\vec{i} + b\vec{j}$ form of the vector with magnitude 4 and direction angle $90°$.

XIII. Use parametric equations.

33. Make a table of points by hand from a set of parametric equations and sketch a graph by hand from the points.

    a. Make a table of points and graph the resulting graph of *x* and *y* if:

       $x = t+1$

       $y = t^2 - 2$

34. Eliminate the parameter and create an equation in *x* and *y*.

    a. Write an equation in *x* and *y* equivalent to the parametric equations.

       $x = t+1$

       $y = t^2 - 2$

XIV. Work with the definitions, equations, and graphs of conic sections.

35. Sketch by hand the graph of the equations of a parabola in the form:

$$x^2 = 4ay \quad or \quad y^2 = 4ax$$

a. Sketch the graph of $y^2 = -12x$.

b. Identify the focal point of $y^2 = -12x$.

c. Give the equation of the directrix of $y^2 = -12x$.

36. Find an equation of a parabola whose vertex is at the origin if the equation of its directrix and its focal point are given.

a. Find an equation of a parabola with focal point $(0,4)$ and directrix $y = -4$.

37. Write an equation of an ellipse or hyperbola when given sufficient information.

a. Write an equation of the ellipse with foci $(0,2)$ and $(0,-2)$ with vertices $(0,3)$ and $(0,-3)$.

b. Write an equation of the hyperbola with foci $(4,0)$ and $(-4,0)$ and vertices $(3,0)$ and $(-3,0)$.

38. Write equations of asymptotes of hyperbolas given their equation or other information.

a. Write the equations of the asymptotes of

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

39. Graph conic sections given their equations using horizontal and/or vertical shifts. Identify center, vertices, foci, asymptotes, and/or directrix:

a. $(y-2)^2 = 8(x+3)$

b. $(x+5)^2 + (y-3)^2 = 4$

c. $\dfrac{(x-2)^2}{4} + \dfrac{(y+3)^2}{9} = 1$

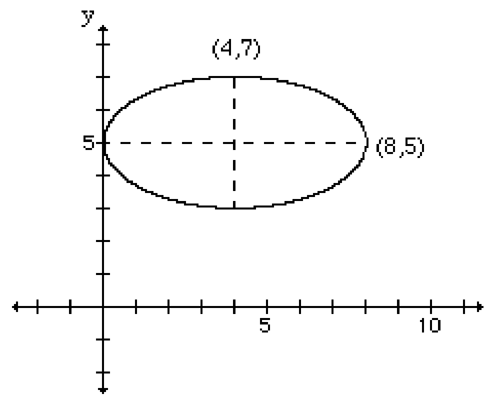
d. $\dfrac{(x-2)^2}{4} - \dfrac{(y+3)^2}{9} = 1$

40. Identify the type of conic section from a given equation and graph it.

a. Identify the type of conic section described by the following equation. Sketch its graph.
$9x^2 + 4y^2 - 36x + 8y + 31 = 0$

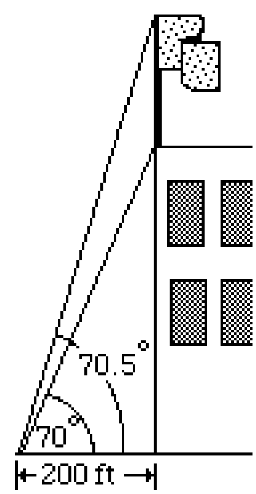
41. Write algebraic equations for conic sections from graphs or other information.

a. Write an equation for the ellipse shown:



XV. Apply geometric and trigonometric relationships to appropriate multi-step problems.

42. From a point 200 feet from the base of a building, the angle of inclination to the base of a flagpole at the edge of the building is 70^o. The angle of inclination to the top of the flagpole is 70.5^o. How tall is the flagpole?



43. A wheel travels 1 mile in 2 minutes. Calculate the angular velocity of the wheel in revolutions per second if the wheel has a diameter of 2.5 feet.

XVI. Use a graphing calculator to graph equations and explore concepts for equations in rectangular, parametric, or polar form.

44. Solve each equation graphically.

    a. $2\sin x = 0.7x$

    b. $\cos x = e^x$ on the interval $-\pi \le x \le \pi$.

45. Find the zeros of the function $f(x) = 2\sin x - 3\cos 2x$ on the interval $0 \le x \le 2\pi$.

46. Find the maximum value of the function in #45 above on the interval $0 \le x \le 2\pi$. Where does this maximum occur?